AIAA Journal
Vol. 49, No. 6, June 2011

Technical Notes

Can Solar Sails Flutter?

Earl H. Dowell*
Duke University, Durham, North Carolina 27707

DOI: 10.2514/1.J050900

I. Introduction

IN A widely cited report, MacNeal [1] has pointed out the advantages of a heliogyro, a solar sail that rotates and responds in ways similar to a helicopter blade, with the driving force being the solar pressure, as distinct from the aerodynamic pressure of more conventional rotorcraft.

In this brief Note we consider the possible interaction between the deformation of the solar sail and the solar pressure. The solar pressure acts normal to the surface of the sail; thus, as the sail deforms, the direction of the solar pressure changes. This provides an opportunity for feedback between the loading of the solar pressure and the deformation of the sail. Indeed, as is shown implicitly by MacNeal [1] and more explicitly here, for small elastic deformations of the sail, the solar-pressure-loading/solar-deformation relationship has essentially the same mathematical form as the aerodynamic-loading/elastic-structural-deflection relationship for (two-dimensional, static Ackeret) supersonic flow aerodynamic theory. This suggests that many of the known methods and, to some extent, the known results from aeroelasticity may be immediately adapted to the analysis of the solar sail. In particular, solar sails have the potential to undergo flutter.

II. Dependence of Solar Pressure on Elastic Sail Deformation

Following MacNeal [1], consider Fig. 1, adapted from Figure 11 of his report. Regarding this figure and quoting MacNeal,

> Figure 11 shows the coordinate geometry for a rotating blade that is coned through an angle β with respect to the plane of rotation and subsequently pitched through an angle θ with respect to the tangent to the cone of rotation. A further rotation, ζ, about the normal to the blade produces no change in the radiation pressure need not be considered. The [solar pressure] illumination lies in the $\bar{x}$, z plane at an angle γ with respect to the axis of rotation.

The following expression is derived in [9] of the report by MacNeal [1] for the normal solar pressure on the blade surface under the assumption that the coefficient of reflectivity is unity. This is MacNeal's equation (82) in [1]. Although the derivation is not given in [1] per se, the result is readily derived and was rederived by the present author by considering the cosine of the angle between the direction of solar illumination and the instantaneous normal to the solar sail. The solar pressure is given by a constant p_0, which depends on the characteristics of the sun and the distance from the sun, multiplied by the square of the cosine of this angle. The result is as follows:

$$p_n = p_0[(\sin\theta\sin\psi - \cos\theta\sin\beta\cos\psi]\sin\gamma + \cos\theta\cos\beta\cos\gamma]^2 \tag{1}$$

(In the derivation of the present author there is a sign change from that of MacNeal [1] with respect to the sun angle, but since all sun angles must be considered in the subsequent analysis, the eventual result is the same.)

MacNeal [1] further notes that this equation can be simplified if β and θ are small angles. Thus, making the usual linear approximations for cosine and sine of β and θ, Eq. (1) becomes the following:

$$p_n = p_0[(\theta\sin\psi - \beta\cos\psi)\sin 2\gamma + \cos^2\gamma] \tag{2}$$

Note further that β and θ can be generalized as the local slopes in the y and x directions for the heliogyro or, indeed, for any other elastic body. That is, β and θ may now be written as

$$\theta = \frac{\partial w}{\partial x} \qquad \beta = \frac{\partial w}{\partial y} \tag{3}$$

where w is the elastic deformation in the direction normal to the surface, which is the z axis for small elastic deformations and, in general, varies over the surface of the solar sail; i.e., w is a function of x and y as well as time t. It is the dependence of the perturbation solar pressure on the local slopes of the elastically deformed solar sail that create an analogy with supersonic flow aerodynamic theory. In the language of aeroelasticity this is a quasi-static theory, as the solar pressure depends on the instantaneous deformation of the solar sail and there is no time-memory effect, since the relevant speed of propagation is the speed of light.

III. Example of Solar Sail Panel Flutter

As an example of the potential for solarelastic flutter, the equation of motion for the solar sail will be taken as that of a rotated (but nonrotating) elastic plate under tension. D is the plate bending stiffness, which is proportional to the solar sail thickness cubed and T_x and T_y are the tensions in the x and y directions, respectively. With the expression for the solar pressure included, the equation of motion becomes the following. This equation is mathematically analogous to that studied extensively for the aeroelasticity of plates and shells [2].

$$D\nabla^4 w - T_x\frac{\partial^2 w}{\partial x^2} - T_y\frac{\partial^2 w}{\partial y^2} + m\frac{\partial^2 w}{\partial t^2} = p_n \tag{4}$$

Note that in Eq. (4), the term in the solar pressure [see Eq. (2)] that does *not* depend on the deformation can be neglected for a *linear* mathematical model. Later, this term will be discussed in the context of a *nonlinear* analysis. The corresponding equation for the aeroelasticity of plates and shells is

$$D\nabla^4 w - T_x\frac{\partial^2 w}{\partial x^2} - T_y\frac{\partial^2 w}{\partial y^2} + m\frac{\partial^2 w}{\partial t^2} = p_a \tag{5}$$

where the aerodynamic pressure p_a is given for two-dimensional supersonic flow by

$$p_\alpha = [2q/(M^2-1)^{1/2}]\frac{\partial w}{\partial \bar{x}}$$

or

Received 2 September 2010; revision received 8 February 2011; accepted for publication 9 February 2011.

*Professor, Mechanical Engineering and Materials Science, P.O. Box 90300. Honorary Fellow AIAA.

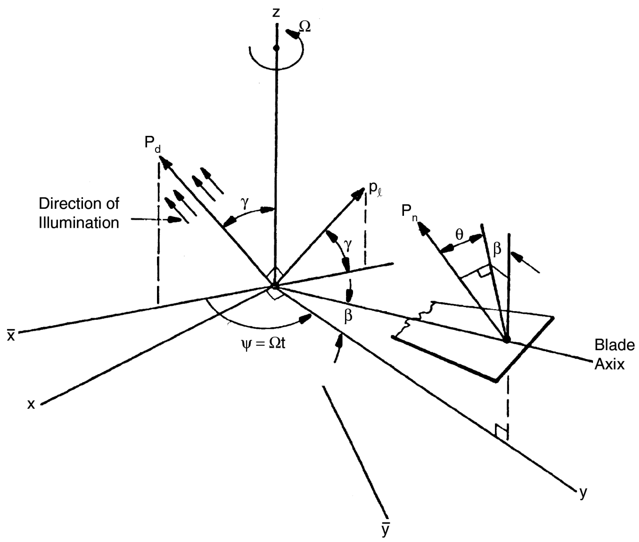

Fig. 1 Coordinate geometry of solar sail in reference to the sun angle.

$$p_a = [2q/(M^2 - 1)^{1/2}]\left[\frac{\partial w}{\partial x}\sin\psi - \frac{\partial w}{\partial y}\cos\psi\right] \tag{6}$$

For convenience and to state the analogy most simply, the aerodynamic flow is aligned with the $\bar{x}$ axis, but in the negative $\bar{x}$ direction. If the flow is aligned in the positive $\bar{x}$ direction, then there is a minus sign introduced in the expression for the aerodynamic pressure. If all edges of a rectangular (or doubly symmetric) plate have the same boundary conditions this minus sign is immaterial, but for other edge conditions and plate geometries, it must be taken into account.

In Eq. (6) q is the dynamic pressure and M is the Mach number. Two-dimensional quasi-static supersonic aerodynamic theory is used in Eq. (6) [2]. To adopt the results for Eqs. (5) and (6) from aeroelasticity to obtain analogous results for Eqs. (2–4) for solar elasticity [compare Eqs. (2) and (6)], one simply replaces

$$2q/(M^2 1)^{1/2} \quad \text{by} \quad p_0 \sin 2\alpha \tag{7}$$

A. Results from Aeroelasticity that Also Apply to Solarelasticity

Without further calculation one can recall some of the results of [2] and the aeroelasticity literature discussed there. For example, it is known that placing the long dimension of the sail parallel to the plane of illumination will give the most critical flutter condition; i.e., it will require the thickest sail to prevent flutter. Conversely, placing the short dimension parallel to this plane will give the least critical flutter condition; i.e., it will give the smallest sail thickness required to prevent flutter. Also, the term in Eq. (2) for the solar pressure that does *not* depend on structural deflection w and was previously neglected may be included in a *nonlinear* structural model. Its effect will be to alleviate flutter; i.e., the thickness required to prevent flutter will be decreased. Hence, neglecting this term and using a *linear* flutter analysis is conservative. Of course, there are other nonlinear effects that might be nonconservative such as the effect of thermal buckling [2], but thermal buckling and other nonlinear effects are beyond the scope of the present Note, although certainly worthy of study. Again, the methodology developed for the aeroelasticity of plates and shells [2] may prove useful for study of the effects of thermal buckling and solarelastic response.

Other parameters of interest include an orthotropic sail versus an isotropic sail, various edge boundary conditions, and sail curvature. All of these have been considered in the aeroelastic literature [2]. Finally, a word about the "membrane paradox" is worthy of mention. If the bending stiffness D is set to zero and a pure membrane model of the sail is used, it is well known from the aeroelastic literature that flutter will not occur. The correct physical interpretation of this result is that if D is finite and T_x and T_y tend to infinity, then the aerodynamic or solar pressure required to create flutter also tends to infinity. On the other hand, if T_x and T_y are finite and D tends to zero, then the aerodynamic or solar pressure required to create flutter is always finite for any nonzero value of D. Mathematically the limit of D tending to zero is a singular perturbation problem and there is a structural boundary layer near the leading and trailing edges of the plate. This has been first discussed by Spriggs et al. [3] and later elaborated upon by Dowell and Ventres [4].

B. Numerical Example

A numerical example may be illuminating. From the aeroelasticity literature it is well known that if a certain nondimensional parameter λ exceeds a critical volume λ_{cr}, then flutter may occur when

$$\lambda_a \equiv \frac{q}{(M^2 1)^{1/2}} a^3/D > \lambda_{\text{cr}} \tag{8}$$

The corresponding solar elastic parameter is

$$\lambda_s \equiv p_0(\sin 2\gamma)a^3/D > \lambda_{\text{cr}} \tag{9}$$

where

$$D \equiv Eh^3/12(1 - v^2) \tag{10}$$

Note that λ_{cr} is the same in both Eqs. (8) and (9).

The typical values from MacNeal [1] are as follows: illumination angle $\sin 2\gamma = 1$, thickness $h = 0.00025$ m, modulus of elasticity $E = 10^6$ psi, and solar pressure magnitude $p_0 = 0.1304 \times 10^{-8}$ psi *or* 0.9×10^{-4} dyne/cm^2.

Now λ_{cr} is of the order of 1000 for a square plate, with the stiffness due to bending and tension of comparable magnitude. Then from Eqs. (9) and (10) it is seen that for solar sails whose lengths are in excess of a kilometer flutter may occur. Note that a recently proposed solar sail is much thinner, i.e., of the order of 7.5 μm, and therefore could flutter with a much smaller length dimension [5]. Of course, by increasing the tension sufficiently, flutter may be suppressed. A heliogyro is one way to do this effectively.

IV. Conclusions

The solar pressure on a deforming body provides a force that depends on that deformation and thus a source of feedback between the force and the deformation that can lead to a dynamic instability or flutter. It is shown that this force has the same mathematical form as for the supersonic fluid flow over a deforming body. Invoking this analogy between supersonic fluid flow and solar pressure, the potential for dynamic instability or flutter of solar sails is assessed.

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E. Livne
Associate Editor